Filters

Discrete-time filter

Filter	$\hat{H}(w)$	$h[n]$
Low-pass	$\hat{H}_\ell(w) = p_{w_c}(w + k2p)$	$\frac{1}{2p}\int_{-w_c}^{w_c} 1\cdot e^{jwn}dw = \frac{\sin(w_c n)}{pn}$
High-pass	$\hat{H}_h(w) = 1 - \hat{H}_\ell(w)$	$d[n] - \frac{\sin(w_c n)}{pn} = \begin{cases} 1-\frac{w_c}{p} & n=0 \\ -\frac{\sin(w_c n)}{pn} & n\neq 0 \end{cases}$
Band-pass $w_2 > w_1$	$\hat{H}_b(w) = p_{w_2}(w) - p_{w_1}(w)$ or $\hat{H}_b(w) = p_{w_c}(w - w_0) + p_{w_c}(w + w_0)$ $w_0 = \frac{w_2 + w_1}{2}$ and $w_c = \frac{w_2 - w_1}{2}$.	$\frac{\sin(w_2 n)}{pn} - \frac{\sin(w_1 n)}{pn}$

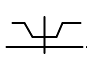
Continuous-time filter

Ideal low-pass filter	Ideal high-pass filter	Ideal band-pass filter
$\hat{H}(w) = p_{w_0}(w)$ $h(t) = \frac{\sin(w_0 t)}{pt}$	$\hat{H}(w) = 1 - p_{w_0}(w)$	$\hat{H}(w) = p_{\frac{w_2-w_1}{2}}\left(w - \frac{w_2+w_1}{2}\right)$ $= p_{w_2}(w)\cdot(1 - p_{w_1}(w))$ $= p_{w_2}(w) - p_{w_1}(w)$
pass band: $\lvert w\rvert < w_0$	$\lvert w\rvert > w_0$	$w_1 < \lvert w\rvert < w_2$
stop band: $\lvert w\rvert > w_0$	$\lvert w\rvert < w_0$	$\lvert\omega\rvert < w_1$ and $\lvert w\rvert > w_2$

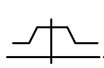
- $w_0 \Rightarrow$ **cut-off frequency**
- Noise reduction via pre-emphasis / de-emphasis

  interesting signal $x(t) \Rightarrow$ low-pass

  noise $n(t) \Rightarrow$ broadband (flat $\hat{N}(w)$)

  1) $\longrightarrow \hat{H}_1(w)$  $\longrightarrow$

  2) $\longrightarrow + \hat{N}(w) \longrightarrow$ record

  3) $\longrightarrow \hat{H}_2(w) = \frac{1}{\hat{H}_1(w)}$  $\longrightarrow$

$$\hat{Y}(w) = \hat{H}_2(w)\left(\hat{H}_1(w)\hat{X}(w) + \hat{N}(w)\right) = \hat{X}(w) + \hat{H}_2(w)\hat{N}(w)$$

y(t) contain less high-frequency content

- Want: $w(t) * h_{desired}(t) = y_{desired}(t)$

$$\hat{H}_{desired}(w) = \left|\hat{H}_{desired}(w)\right| \underset{\Im^{-1}}{\overset{\Im}{\rightleftharpoons}} h_{desired}(t)$$

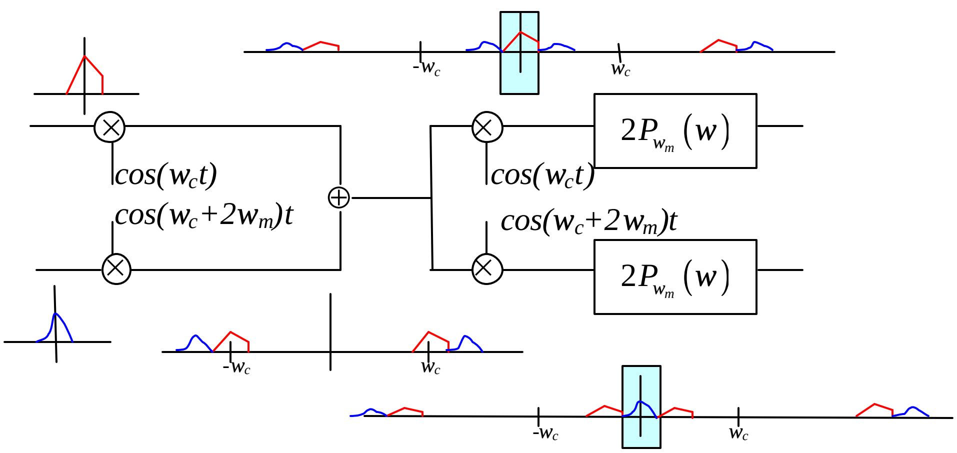
Suppose can build

Linear phase $\Rightarrow$ $f_{\hat{H}}(w) = -wt_1$

$$\hat{H}(w) = \left|\hat{H}_{desired}(w)\right| e^{-jwt_1} \underset{\Im^{-1}}{\overset{\Im}{\rightleftharpoons}} h(t) = h_{desired}(t - t_1)$$

$$\left.\begin{aligned} \hat{Y}(w) &= \hat{W}(w)\cdot\hat{H}(w) \\ &= \left(\hat{W}(w)\hat{H}_{desired}(w)\right)e^{-jwt_1} \\ &= \hat{Y}_{desired}(w)e^{-jwt_1} \end{aligned}\right\} \underset{\Im^{-1}}{\overset{\Im}{\rightleftharpoons}} y(t) = y_{desired}(t - t_1)$$

Frequency-division multiplexing



Amplitude Modulation

- To transmit low-pass signal over a high-pass channel

AM with synchronic demodulation

- w_c = **carrier frequency** >> $2\omega_m$
- Encode

$$z(t) = x(t)\times\cos(w_c t) = \frac{1}{2}x(t)e^{jw_c t} + \frac{1}{2}x(t)e^{-jw_c t}$$

$$\hat{Z}(w) = \frac{1}{2}\hat{X}(w - w_c) + \frac{1}{2}\hat{X}(w + w_c)$$

Proof Use Frequency-shift/modulation rule: $e^{jw_1 t}x(t) \underset{\Im^{-1}}{\overset{\Im}{\rightleftharpoons}} \hat{X}(w - w_1)$.

- $x(t)\times\cos(w_c t) \underset{\Im^{-1}}{\overset{\Im}{\rightleftharpoons}} \frac{1}{2}\hat{X}(w-w_c)+\frac{1}{2}\hat{X}(w+w_c)$

- Decode (Demodulation)
  - $g(t)=z(t)\times\cos(w_c t)$

  $$\begin{aligned}\hat{G}(w)&=\frac{1}{2}\hat{Z}(w-w_c)+\frac{1}{2}\hat{Z}(w+w_c)\\&=\left(\frac{1}{4}\hat{X}(w-2w_c)+\frac{1}{4}\hat{X}(w)\right)+\left(\frac{1}{4}\hat{X}(w)+\frac{1}{4}\hat{X}(w+2w_c)\right)\\&=\frac{1}{4}\hat{X}(w-2w_c)+\frac{1}{2}\hat{X}(w)+\frac{1}{4}\hat{X}(w+2w_c)\end{aligned}$$

  - $y(t)=g(t)*h(t)\ ;\widehat{H}(w)=2P_{w_m}(w)$ (low-pass)

  $$\hat{Y}(w)=2P_{w_m}(w)\cdot\hat{G}(w)=\hat{X}(w)$$

- need exactly the same cos(ω_ct) at both transmitter and receiver

AM with asynchronous demodulator

- Encode

$f(t)=1+m\cdot x(t)\ ;\begin{cases}\text{choose small enough m (modulation index)}\\ \text{that make } f(t) > 0, \forall t\end{cases}$

For $\min(x(t))<0$, choose $m<\frac{-1}{\min(x(t))}$

$$\begin{aligned}z(t)&=f(t)\times\cos(w_c t)=(1+m\cdot x(t))\times\cos(w_c t)\\&=\cos(w_c t)+m\cdot x(t)\cdot\cos(w_c t)\end{aligned}$$

$$\hat{Z}(w)=(pd(w-w_c)+pd(w+w_c))+\frac{m}{2}\left(\widehat{X}(w-w_c)+\widehat{X}(w+w_c)\right)$$

- Decode

$z(t)\xrightarrow{\text{peak detector}} f(t)$

$x(t)=\frac{1}{m}(f(t)-1)$

- Use lots of transmitter power to send *cos(*w_c*t)*-term in *z(t)*

Time-division multiplexing

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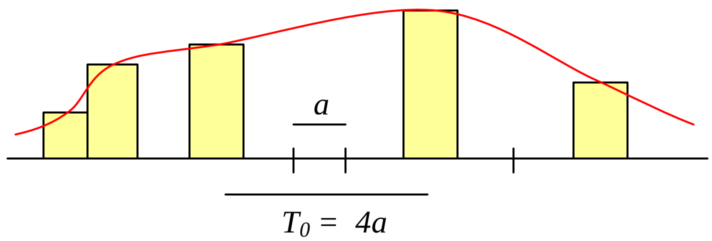

Use $T_0 = 4a$ (Use the max. separation)